بسم الله الرحمن الرحيم

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Numerical Methods for ODEs

MATH 622

الفصل الدراسي الاول لعام ٢٠١٦

**Student’s Initial Knowledge**

The student who is enrolled in this course must have had at least grade B in the following

undergraduate courses:

1. Numerical Analysis II (MATH 423): To make sure the student had covered topics of basic programming of ODE using Fortran, Maple, MatLab or Mathematica softwares

2. Linear Algebra I (MATH 241): To make sure the student has covered the topics of linear algebra such as linear systems and matrices with all the necessary techniques to solve the eigenvalue problem

3. Differential Equation II (MATH 305): To make sure the student has covered the topics of solving ODE analytically using the different methodologies.

**Aim and Objectives:**

To provide the student with the required computational knowledge to study and evaluate the

solution of the differential equations (initial and boundary value problem-IVP and BVP) as

well as the integral equation. The relation between Mathematics and other fields of science is

given as examples.

Course flow:

The course will cover the topics that will enable the student to:

1. Compute the approximated solution of the IVP and the error analysis related to each technique with handling the different types of error involved, which will enable her to choose the best technique.

2. Studying the IVP system, analyzing it and studying its stability

3. Compute the approximated solution of the BVP

4. Compute the exact and approximated solution of the Integral equations

**Course Design**

This course is designed to:

1. Guide the student into a more interpretive mode of thinking

2. Use any mathematical-software (in our case Maple is used) to enhance the ability to compute symbolic and numerical solutions, and to perform qualitative and graphical analysis to differential equation (DE)

3. Develop the student’s skills to solve the applications of DE.

4. Enhance the student’s ability to choose a problem in DE, analyse it, present it and discuss

it.

5. Introduce the approximated and exact solutions for both BVP and integral equations (IE).

**Obstacle of this term:**

1-Two of the students have no idea about programming, especially using Maple or Matlab. A basic introduction will be a benefit and will leave the rest to them.

2-The Maple in the lab is not working properly. This happened after the new PCs were installed.

3-The faculty decided to use the lab for the other departments’ meetings and the PG vivas, this will effect the students’ use of the labs. Seminars and training courses were also run in the labs.

**Course’s Brief:**

The first section of this course is designed to solve and analyze the ODE. The student will learn from scratch how to draw a flowchart and write a program to solve problems using Maple programming, such as, solving ODE numerically, analytically and handling the error that arise. The books used are [1] and [2]. The first and higher order of ODE (after transform it into system of ODE) is solved numerically using first, second and fourth order method such as Euler, Taylor, Runge-Kutta, and Predictor-corrector Adam-Bashforth and Adam Moulton. The error involved is analyzed and discussed numerically. The used reference is the book by Burden and Faires[1], chapter(5).

At the end of this section, the student will make a presentation of her work in a seminar and then be evaluated by me.

The second section is devoted to the BVP (boundary value problem) a method (or two) in solving the BVP will be studied together with the error discussion part(s). The same reference[1] of the previous section will be used here, chapter(11) sections(1-3)

The third section is devoted to evaluate the solutions of the IE (integral equations), Fredholm and Volterra mainly. At the beginning of the course, an introduction of the relation between IVP-BVP and IE, then the exact solution using different methods. The reference of this section is the book by Wazwaz[2], chapters(1-3).

A help in programming by Maple can be found in references[3-4]. These two references took the student from A-Z in using Maple as a tool to find solutions (rarely-exact solution in nature but numerical computations is so easy to find the approximated solution) of (mostly) any mathematical problem. Matlab is a preferable tool too, if the student is good in using it, but it won’t deal with symbolic evaluations like Maple.

Main References:

**[1] Numerical Analysis, Richard L. Burden and J. Douglas Faires, Brooks/Cole Thomson Learning (8th or 9th Eds.).**

**[2] A first Course in Integral Equations, Abdul-Majid Wazwaz, World Scientific, USA.**

Lab references

**[3] Dynamic Systems with Applications using Maple, Stephen Lynch, Birkhäuser Boston 2001.**

**[4] Differential Equations with Maple, Kevin R. Coombes, Brian R. Hunt, Ronald L.**

**Lipsman, John E. Osborn and Garrett J. Stuck, University of Maryland at College Park, John Wiley & Sons, Inc**

Course’s Details

Section(1), the book used is [1]

(1)Introduction to understand the meaning of differential equation. The pendulum problem and the representation of it mathematically is given and explained via an initial-value problem (IVP).

(2)Theory of IVP, section 5.1

HW(1): Picard’s method (Page 256 No 9)-by calculator.

(3)Euler’s Method, section 5.2

(4)Drawing the Flowchart of Euler’s

(5)Basics of programming using Maple

(6)Solving ODE and IVP analytically using Maple

(7)Solving the IVP numerically using Maple by Euler’s method

(8)Using the Maple-command printf to control the format of the output,

(9)Higher-Order Taylor’s Methods, section 5.3

HW(2): Types of errors

Project(1): The project will cover section(5.5) of reference[1]

Date of handing Wed 17/12/1436H=30/9/2015

It must be:

1-Typed in TeX and covers detailed explaination of the content and proofs included.

2-Flowchart of the method

3-Presentation in PowerPoint (or better Beamer)

4-Maple program

Quiz(1): This exam will cover both sections (5.1- 5.3)

(10)Runge-Kutta (RK) Method, section 5.4

(a)Driving Midpoint method

(b)Modified Euler method

(c )Heun’s method

(d)RK method

(e)types of error involved

(11)Drawing a flowchart evaluating the approximated solution using RK method

(12)writing a program that produce exact and approximated solution using RK method

HW(3): Pages(282-283) No. 28+30+31

(13)Multistep Methods section 5.6

Project(2): Lecturing the introduction of chapter (5.6) up-tp example(3), using Power-point or better BEAMER.

The methods AB, AM and PC in chapter (5.6) together with the example(14)

HW(4) writing a Maple program to evaluate exercise(7b) using AB4, AM4 and PC, together with the flowchart.

(15)Higher order equations and System of DEs, section 5.9

(a)Introduction to Higher-Order and System of nonlinear DEs, section 5.9

(b)Logical steps and flowcharting of RK method to solve system of ODEs

(c) Writing a Maple program solving a system of ODEs

HW(5): Solve exercise(3d) by hand and by Maple using algorithm 5.7.

Project(3):Write a seminar to be given to Maths Students that

Find the analytical (if any) and numerical solutions for the Prey\_Predator problem in

 exercise(9) in section 5.9

Graph it and discuss the error involved

A handout of the presentation, introduction, program, graph and the analysis has to be handed

Chapter(11): the student will provide us with an introduction of BVP in real life (see the (introduction

Linear shooting method

HW6 problem set 11.1 ex1b, 4b and 6

Project4 The shooting method for nonlinear problems section 11\_2

Finite Difference method for linear proble

HW7:exercise set 11.3 number 3d